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The Principle of Equivalence at Finite Temperature*+

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We demonstrate that the equivalence principle is violated by radiative corrections to the gravitational and inertial masses at finite temperature. We argue that this result can be attributed to the Lorentz noninvariance of the finite temperature vacuum.

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One of the cornerstones of general relativity is the principle of equivalence[1], which states, in its weak form, that the gravitational acceleration is the same for all bodies, or that the inertial and gravitational masses are equal. There is remarkably strong experimental support for this principle[2], with no violation detected at the level of a part in 10^{12}. Since in a quantum theory a portion of a particle's mass (formally infinite!) arises due to quantum radiative corrections, these must also obey the equivalence principle. Explicit calculations demonstrate this to be true[3]. However, in a theory at a non-zero temperature, additional contributions to particle masses arise through finite temperature radiative corrections. It has not previously been determined whether such terms satisfy the equivalence principle, nor is it clear that they should (see below). In this paper we present evidence that the equivalence principle is in fact not valid at nonzero temperature, and offer an explanation of why this violation occurs.

We shall study the electron's mass in finite temperature Quantum Electrodynamics with $0 \leq T << m_e$. The restriction to low temperatures is made primarily to ease the interpretation of the results. At low temperature only the photons form a singificant background heat bath, with the effects of the electron-positron heat bath being exponentially suppressed ($e^{-m_e/T}$). The interpretation of electron motion in an electron-positron sea could be clouded by the Pauli principle. However, the motion of a charged particle in a sea of photons can be clearly observed via standard techniques.

At non-zero temperature, the particle propagators must be modified to take into account the effects of motion through the background heat bath. Thus, for example, in the "real time" formulation of finite temperature field theory, the photon propagator becomes (in Lorentz gauge)[4]

$$D_{\mu\nu}(k) = g_{\mu\nu}[\frac{-i}{k^2+i\varepsilon} - 2\pi\delta(k^2)n_B(k)] \qquad (1)$$

with $n_B(k)$ being the Bose Einstein distrubtion function

$$n_B(k) = \frac{1}{e^{\beta k} - 1} \quad . \tag{2}$$

The last terms in $D_{\mu\nu}$ describes the emission and absorption of real photons from the heat bath. Although calculations of finite temperature effects are simplest in this real time formalism, since the $T = 0$ and $T \neq 0$ components are nicely separated, we have also performed the following calculations via the imaginary time method[5], where the propagators involve a discrete summation in the "energy" variable instead of an integration.

The study of the fermion self energy illustrates some of the novel features of finite temperature field theory. A simple calculation yields

$$\Sigma(p) = \frac{\alpha}{4\pi^2} \{ I_A(\not{p} - m) + \not{I} + \not{L}(p^2 - m^2) + \ldots \} \tag{3}$$

where

$$\begin{aligned} I_A &= 8\pi \int \frac{dk}{k}\, n_B(k) \\ I_\mu &= 2 \int \frac{d^3k}{k_0}\, n_B(k)\, \frac{(k_0, \underset{\sim}{k})}{\omega_p k_0 - \underset{\sim}{p} \cdot \underset{\sim}{k}} \\ L_\mu &= -\frac{1}{\omega_p} \int d^3k\, n_B(k)\, \frac{(k_0, \underset{\sim}{k})}{(\omega_p k_0 - \underset{\sim}{p} \cdot \underset{\sim}{k})^2} \end{aligned} \tag{4}$$

with

$$\omega_p = \sqrt{\vec{p}^{\,2} + m^2} \quad .$$

The standard decomposition into a Lorentz invariant mass shift δm and a wave-function renormalization proportional to $(\not{p} - m)$ is not obtained. Instead there appear noncovariant terms in the self energy. Lacking such co-variance, one must define what is meant by the term "mass" very carefully. One possible definition of a particle's mass, which we will call the "phase space mass", is given by the location of the pole in the propagator[6,7]. This occurs at

$$E^2 = \vec{p}^{\,2} + m_p^2$$

with

$$m_p^2 = m^2 + 2\,\frac{\alpha}{4\pi^2}\,p\cdot I \approx \left(m + \frac{\alpha\pi}{3}\,T^2/m\right)^2 \qquad (5)$$

and m being the renormalized mass at T=0. The wavefunction renormalization constant can be determined by requiring that the fields are properly normalized[6] or by requiring that the charge vertex be correctly obtained (see below) and is given by

$$Z_2^{-1} = 1 - \frac{\alpha}{4\pi}\left(\frac{3}{\varepsilon} - 4\right) - \frac{\alpha}{4\pi^2}\left(I_A + 2p\cdot L\right) \qquad (6)$$

where we have included the dimensionally regularized T=0 part (with $1/\varepsilon = 2/(d-4) + \gamma - \ell n\, 4\pi + \ell n\, m^2$) for later use. The mass shift has the physical interpretation of being the effective extra inertia generated by the continual interaction of the electron with the photons in the heat bath.

Note that this "phase space mass" is usually not included in a discussion of the different types of mass. However it can be given a clear operational definition in terms of threshold and phase space behavior for particle reactions. For example a decay of a neutral boson (H^0) into an e^+e^- pair cannot take place if the H^0 mass is below $2m_p$, even if m_{H^0} is larger than 2m . One can thus imagine measuring this phase space mass by looking for the threshold of various reactions. It could also be measured by a careful study of phase space distributions of a particular reaction. Both techniques are presently being utilized in the search for the possible existence of neutrino mass. In principle, the phase space mass can be distinct from either the inertial or gravitational masses. However in our calculation, presented below, it is identical to the inertial mass in the nonrelativistic limit. As an aside, we note that a fourth definition of mass is sometimes used in particle physics studies of chiral symmetry breaking. This "chiral" mass is defined to be the part of the inverse propagator which commutes with the Dirac matrix γ_5 ($\not p$ anticommutes with γ_5). At finite temperature this is equal to the T=0 mass m and <u>not</u> the phase space mass. At T=0, all four definitions of mass coincide, but this is not true at finite temperature, and one of the motivations of this work is to decide which definition of the mass is appropriate to the gravitational coupling.

In order to define the inertial mass operationally, we imagine applying an electric field and studying the consequent acceleration of a charged particle. To do this correctly, we must evaluate the finite temperature corrections to the electromagnetic vertex. The relevant diagrams are given in figure 1 and a straightforward calculation yields

$$\Gamma_\mu = \gamma_\mu\left(1 - \frac{\alpha}{4\pi^2} I_0/E\right) + \frac{\alpha}{4\pi^2} I_\mu \quad . \tag{7}$$

That this result corresponds to no net change in the electromagnetic vertex can be seen in two ways. When the vertex is sandwiched between spinors, $u_\beta(p)$, which take into account the finite T self energy

$$[\not p - m \; - \Sigma(p)]u_\beta(p) = 0 \quad , \tag{8}$$

one obtains

$$e\bar{u}_\beta(p)\Gamma_\mu \, u_\beta(p) = e\,\frac{p_\mu}{E} \quad , \tag{9}$$

i.e. the form of the vertex exactly compensates for the non-covariant self energy in order to maintain the physical current unchanged. Equivalently one can examine the non-relativistic limit. Since the Dirac wavefunction should satisfy (removing the wavefunction renormalization)

$$(\not p - m \; - \frac{\alpha}{4\pi^2} \not I)\psi = e\Gamma_\mu A^\mu \psi \tag{10}$$

we can determine the nonrelativistic Hamiltonian by means of a Foldy-Wouthuysen transformation, which yields a Schrodinger equation of the form

$$i\partial_t \psi_s = [m + \frac{2\alpha\pi T^2}{3m} + \frac{\vec{p}^{\,2}}{2(m + \frac{\alpha\pi T^2}{3m})} + e\phi + \frac{\vec{p}\cdot\vec{A} + \vec{A}\cdot\vec{p}}{2(m + \frac{\alpha\pi T^2}{3m})} + \ldots]\psi_s \tag{11}$$

In order to formally identify the inertial mass we apply an electric field and calculate the acceleration

$$\vec{a} = [H,[H,\vec{r}]] = \frac{e\vec{E}}{m + \frac{\alpha\pi T^2}{3m}} \qquad \text{or} \qquad m_I = m + \frac{\pi\alpha T^2}{3m} \quad . \tag{12}$$

Thus the inertial mass is equal to the phase space mass in this limit, and is increased by temperature corrections. Physically this makes sense, as it represents the increased inertia needed to travel through the background heat bath. As also might be expected, this effect is smaller for more massive particles than it is for light ones.

We now turn to the gravitational coupling. We will work in the weak field limit, i.e. to first order in the gravitational field with the radiative corrections being calculated in flat space. For this purpose we need to study the renormalization of the energy momentum tensor

$$T_{\mu\nu} = \frac{\partial\sqrt{-g}L}{\partial g_{\mu\nu}} = \bar{\psi}[\frac{i}{2}(\gamma_\mu \nabla_\nu + \gamma_\nu \nabla_\mu) - g_{\mu\nu}(i\nabla\!\!\!/ - m^{(u)})]\psi$$

where $m^{(u)}$ is the unrenormalized mass, related to the renormalized T=0 mass, m , using first order dimensional regularization, by

$$m = m^{(u)}[1 - \frac{\alpha}{4\pi}(\frac{3}{\varepsilon} - 4)] \quad .$$

The relevant diagrams are indicated in Fig. 2. In this case we shall quote the results for both the T=0 renormalization and the finite T correction for each diagram. We find for the particle's energy-momentum vertex:

$$\text{Fig. 2a} = -\frac{\alpha}{4\pi}\{(\frac{\gamma_\mu p_\nu + \gamma_\nu p_\mu}{2})(\frac{7}{3\varepsilon} - \frac{56}{9}) - m\, g_{\mu\nu}(\frac{4}{3\varepsilon} - \frac{29}{9})$$

$$+ \frac{1}{\pi}[(\frac{\gamma_\mu p_\nu + \gamma_\nu p_\mu}{2}) I_A + mJ_{\mu\nu} - \frac{p_\mu I_\nu + p_\nu I_\mu}{2m} + g^{\mu\nu}\frac{4\pi^3 T^2}{3m}]\}$$

$$\text{Fig. 2b} = -\frac{\alpha}{4\pi}\{(\frac{\gamma_\mu p_\nu + \gamma_\nu p_\mu}{2})(\frac{8}{3\varepsilon} - \frac{34}{9}) - mg_{\mu\nu}(\frac{2}{3\varepsilon} + \frac{2}{9})$$

$$+ \frac{1}{\pi}[-mJ_{\mu\nu} + \frac{p_\mu I_\nu + p_\nu I_\mu}{m} - \frac{4\pi^3 T^2}{3m}(g_{\mu\nu} - 2\delta_{\mu 0}\delta_{\nu 0})]\}$$

$$\text{Fig. 2c} = -\frac{\alpha}{4\pi}\{(\frac{\gamma_\mu p_\nu + \gamma_\nu p_\mu}{2})(-\frac{2}{\varepsilon} + 6) + mg_{\mu\nu}(\frac{5}{\varepsilon} - 7)$$

$$- \frac{1}{\pi}[(\frac{p_\mu I_\nu + p_\nu I_\mu}{m}) + g^{\mu\nu}\frac{4\pi^3 T^2}{3m}]\} \qquad (14)$$

$$\text{Fig. 2d} = -\frac{\alpha}{4\pi}(\frac{\gamma_\mu p_\nu + \gamma_\nu p_\mu}{2})\{(-\frac{6}{\varepsilon} + 8) + \frac{1}{\pi}[-2I_A + 2I_0/E]\}$$

with

$$J_{\mu\nu} = \int \frac{d^3k}{k}\frac{k_\mu k_\nu}{(p\cdot k)^2} \quad .$$

Including Z_2^{-1} for wavefunction renormalization (see Eq. 6) this leads to the renormalized tensor

$$\tau_{\mu\nu} = (\frac{\gamma_\mu p_\nu + \gamma_\nu p_\mu}{2})[1 - \frac{\alpha}{4\pi^2}I_0/E] - \frac{2\alpha\pi T^2}{3m}\delta_{\mu 0}\delta_{\nu 0}$$

$$+ \frac{\alpha}{4\pi^2}(\frac{p_\mu I_\nu + p_\nu I_\mu}{2m}) - g_{\mu\nu}(\not{p} - m - \frac{\alpha\pi T^2}{3m}) \quad . \qquad (15)$$

If one temporarily ignores the temperature dependent terms, this demonstrates that radiative corrections do respect the equivalence principle at zero temperature, as the energy momentum tensor is not modified and the mass which occurs here, m_0, is the same as that which occurs in the propagator. However the finite temperature correction does not have this feature. One way to see this is to take this matrix element between the finite temperature spinors defined above, yielding

$$\bar{u}_\beta(p)\,\tau_{\mu\nu}u_\beta(p) = \frac{p_\mu p_\nu - \frac{2\alpha\pi}{3}T^2\delta_{\mu 0}\delta_{\nu 0}}{E} \qquad (16)$$

Thus a non-covariant temperature-dependent component appears. Perhaps more convincing is to perform a nonrelativistic reduction. Including the gravitational coupling, we have the Dirac equation

$$(\not{p} - m - \frac{\alpha}{4\pi^2} \not{I})\psi = \frac{1}{2} h_{\mu\nu} \tau^{\mu\nu} \psi \tag{17}$$

with

$$h_{\mu\nu} = \begin{pmatrix} 2\phi_g & 0 & 0 & 0 \\ 0 & 2\phi_g & 0 & 0 \\ 0 & 0 & 2\phi_g & 0 \\ 0 & 0 & 0 & 2\phi_g \end{pmatrix} \tag{18}$$

and ϕ_g being the gravitational potential. The Schrodinger equation which emerges from the Foldy-Wouthysen transformation is

$$i\partial_t \psi = [m + \frac{\alpha\pi T^2}{3m} + \frac{\vec{p}^{\,2}}{2(m + \frac{\alpha\pi T^2}{3m})} + (m - \frac{\alpha\pi T^2}{3m})\phi_g]\psi \quad . \tag{19}$$

Computing the acceleration yields the gravitational mass

$$\vec{a} = \frac{m - \frac{\alpha\pi T^2}{3m}}{m + \frac{\alpha\pi T^2}{3m}} \vec{\nabla}\phi_g \quad .$$

$$\text{or} \quad m_g = m - \frac{\alpha\pi T^2}{3m} \tag{20}$$

which is clearly different from the inertial mass (cf. eqn. 12). At finite temperature then the acceleration in a gravitational field is <u>different</u> for particles of different mass. This would in principle yield a violation of the equivalence principle in an Eötvös type experiment, although at accessible temperatures the effect is small. Thus for an electron at 300°K

$$\frac{\alpha\pi T^2}{3m^2} = 2 \times 10^{-17} \tag{21}$$

yielding an unmeasurable effect.

Although perhaps surprising on the surface, this result is not in conflict with the fundamental ideas of gravitational theory. It appears to occur because of the lack of Lorentz invariance of the finite temperature vacuum. We note that if one had used a noncovariant cutoff (instead of dimensional regularization) in the calculation of the standard T=0 radiative corrections, one would not have obtained an answer consistent with the equivalence principle. This is known from work on the trace anomaly for the energy-momentum tensor, as the trace anomaly is required for the equivalence principle[3]. The fundamental ideas which led to the equivalence principle include the impossibility of defining absolute motion through the vacuum and the indistinguishability of acceleration and gravitational force. However, one can measure absolute velocity and acceleration relative to the heat bath (as has been done for the velocity of the Earth in the 3°K photon distribution left over from the early universe). Thus the conditions under which the equivalence principle was formulated are not met at finite temperature. The fact that we do live in a universe at a nonzero temperature could in principle have led to unexpected results in the Eötvös experiments if it were not for the fact that the correction is too small to be detected at present temperatures. However, we may ascribe this result not to any intrinsic violation of the equivalence principle in the fundamental Hamiltonian describing the universe, but rather to the particular physical state in which we exist.

These results reinforce the connection between Lorentz covariance (or more properly general covariance) and the equivalence principle, but also serve to demonstrate that there are physically realizable situations in which the equivalence principle is not satisfied.

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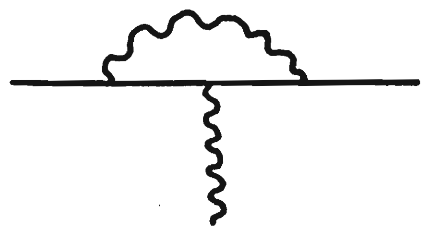
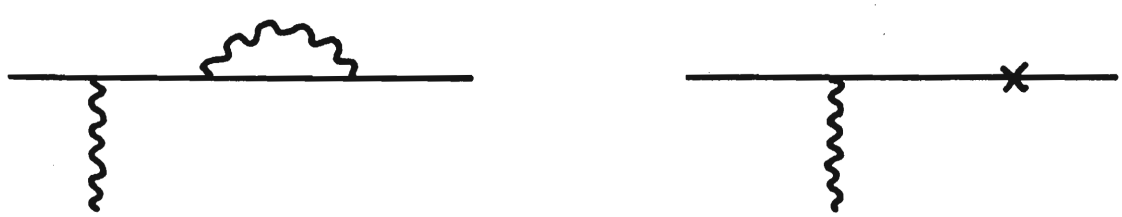


(a)



(b)

Figure 1

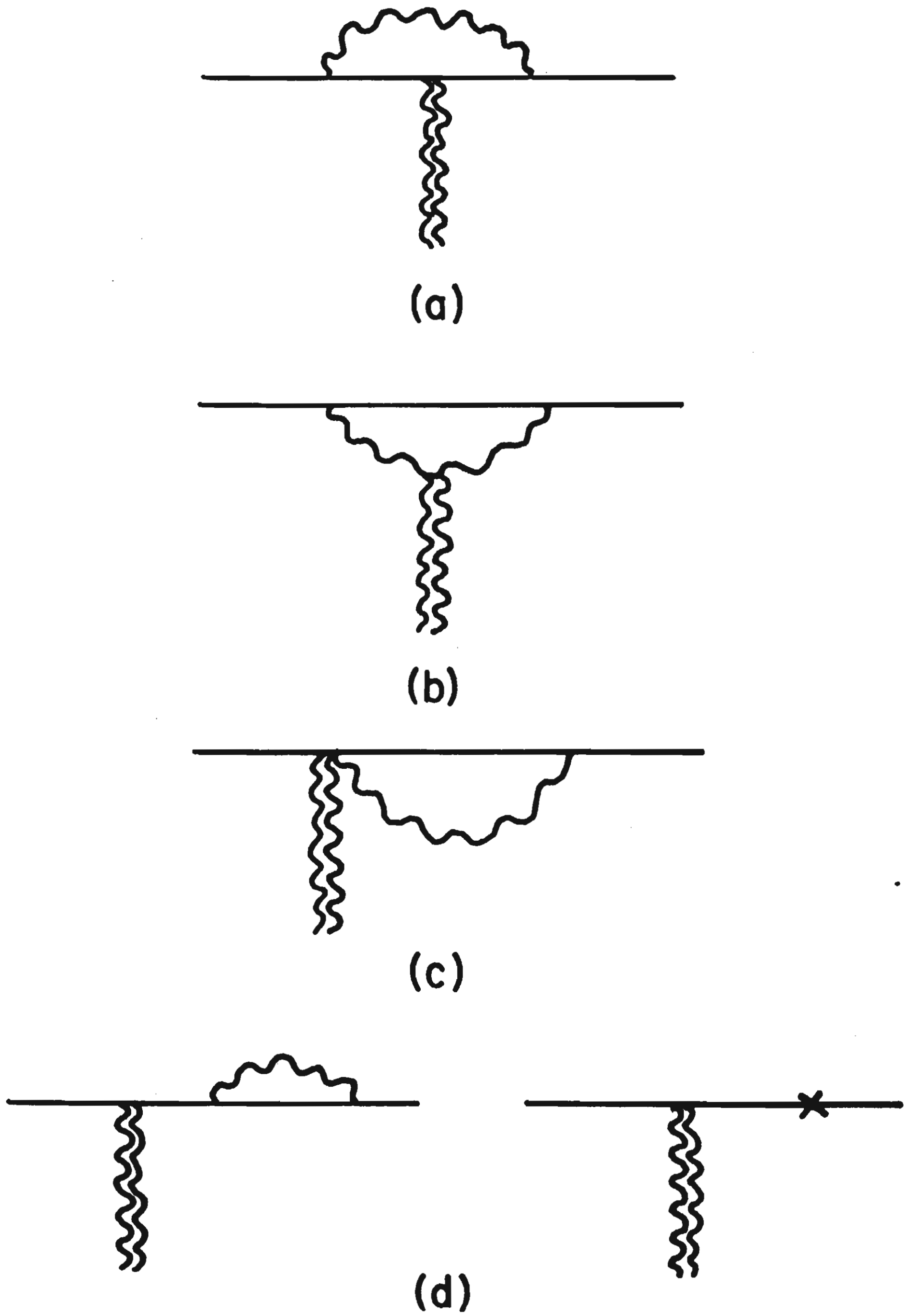


Figure 2